08 — Change over Time

Kieran Healy

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Change over Time

Load our libraries

```r
library(here)       # manage file paths
library(socviz)     # data and some useful things, especially %nin%
library(tidyverse)  # your friend and mine

library(scales)     # Convenient scale labels

## New packages
# install.packages("tsibble") # Time series objects
# install.packages("feasts")  # Time series feature analysis
# install.packages("slider")  # Moving averages and related methods
# remotes::install_github("kjhealy/demog") # Some US demographic data

library(tsibble)
library(feasts)
library(slider)
library(demog)
```

A Time Series: US Monthly Births

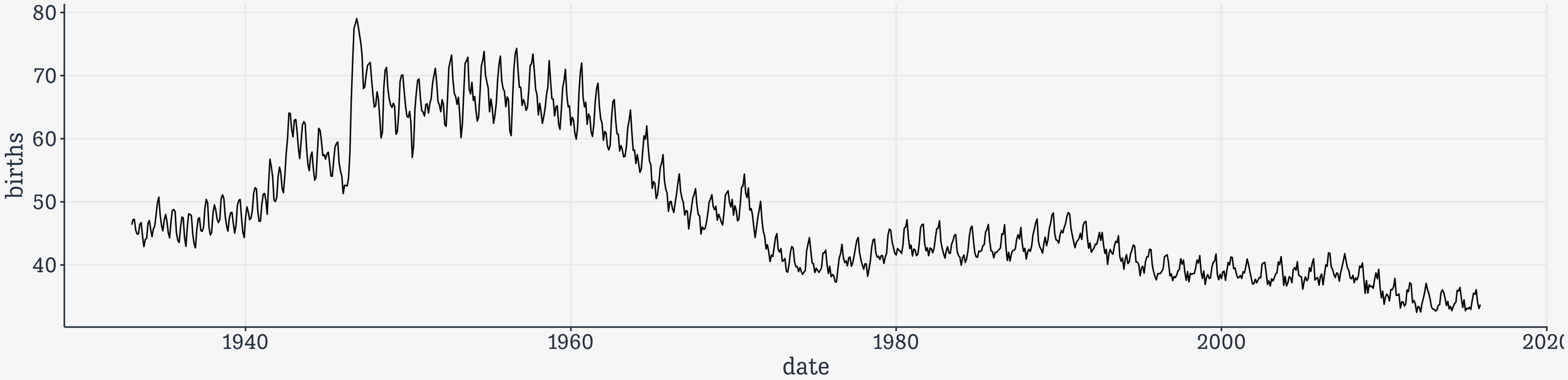
```r
boom <- okboomer |>
  filter(country == "United States") |>
  select(date, total_pop, births_pct_day) |>
  rename(births = births_pct_day)

boom
```

```
# A tibble: 996 × 3
   date        total_pop births
   <date>          <dbl>  <dbl>
 1 1933-01-01 125579000   46.4
 2 1933-02-01 125579000   47.2
 3 1933-03-01 125579000   47.2
 4 1933-04-01 125579000   45.5
 5 1933-05-01 125579000   44.9
 6 1933-06-01 125579000   44.9
 7 1933-07-01 125579000   46.5
 8 1933-08-01 125579000   46.7
 9 1933-09-01 125579000   44.5
10 1933-10-01 125579000   42.9
# ℹ 986 more rows
```

Here the `births` column means "Average daily births per million population"

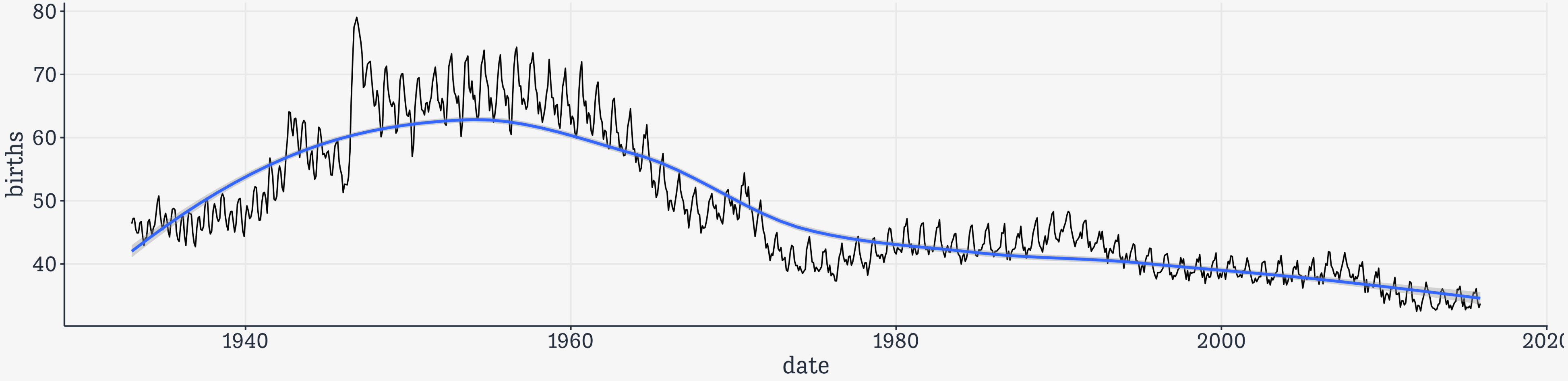
Looking at the Series

```
boom |>
  ggplot(mapping = aes(x = date,
                       y = births)) +
  geom_line()
```



Looking at the Series

```
boom |>
  ggplot(mapping = aes(x = date,
                       y = births)) +
  geom_line() +
  geom_smooth()
```



Too much smoothing here

Time Series Decomposition

The analysis of Time Series is a big area; people often want to see into the future

We will focus on a couple of elementary methods that are more purely descriptive, particularly the idea of *decomposing* a time series into its *trend*, *seasonal*, and *remainder* components.

Decomposition methods are descriptive rather than predictive. They also make assumptions about the character of the data (e.g. its seasonality) which might be something we want to investigate.

More complex forecasting methods are either more detailed, or attempt to be proper models, or both.

Centered Moving Averages: slider

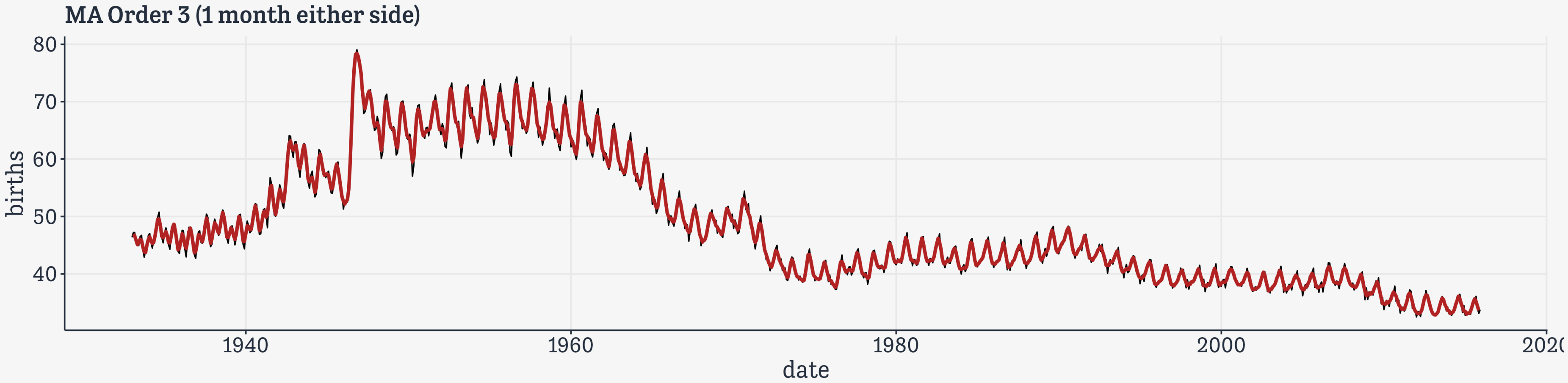
```r
boom |>
  mutate(
    ma3 = slide_dbl(births, mean,
                    .before = 1, .after = 1,
                    .complete = TRUE))
```

```
# A tibble: 996 × 4
   date        total_pop births   ma3
   <date>          <dbl>  <dbl> <dbl>
 1 1933-01-01 125579000   46.4   NA  
 2 1933-02-01 125579000   47.2   46.9
 3 1933-03-01 125579000   47.2   46.6
 4 1933-04-01 125579000   45.5   45.9
 5 1933-05-01 125579000   44.9   45.1
 6 1933-06-01 125579000   44.9   45.4
 7 1933-07-01 125579000   46.5   46.0
 8 1933-08-01 125579000   46.7   45.9
 9 1933-09-01 125579000   44.5   44.7
10 1933-10-01 125579000   42.9   43.8
# ℹ 986 more rows
```

Centered Moving Averages: slider

```r
boom |>
  mutate(
    ma3 = slide_dbl(births, mean,
                    .before = 1, .after = 1,
                    .complete = TRUE)) |>
  ggplot(aes(x = date, y = births)) +
  geom_line() +
  geom_line(aes(x = date, y = ma3), linewidth = rel(1.2), color = "firebrick") +
  labs(title = "MA Order 3 (1 month either side)")
```

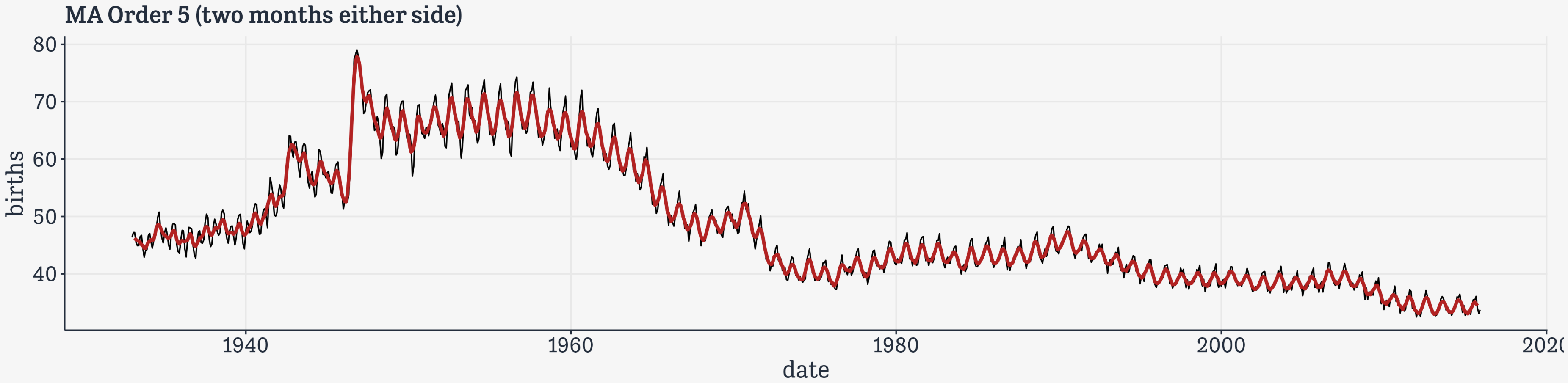
Centered Moving Averages: `slider`



A Centered Moving Average, order 5

```
boom |>
  mutate(
    mav = slide_dbl(births, mean,
                    .before = 2, .after = 2,
                    .complete = TRUE)) |>
  ggplot() +
  geom_line(aes(x = date, y = births)) +
  geom_line(aes(x = date, y = mav), linewidth = rel(1.2), color = "firebrick") +
  labs(title = "MA Order 5 (two months either side)")
```

A Centered Moving Average, order 5

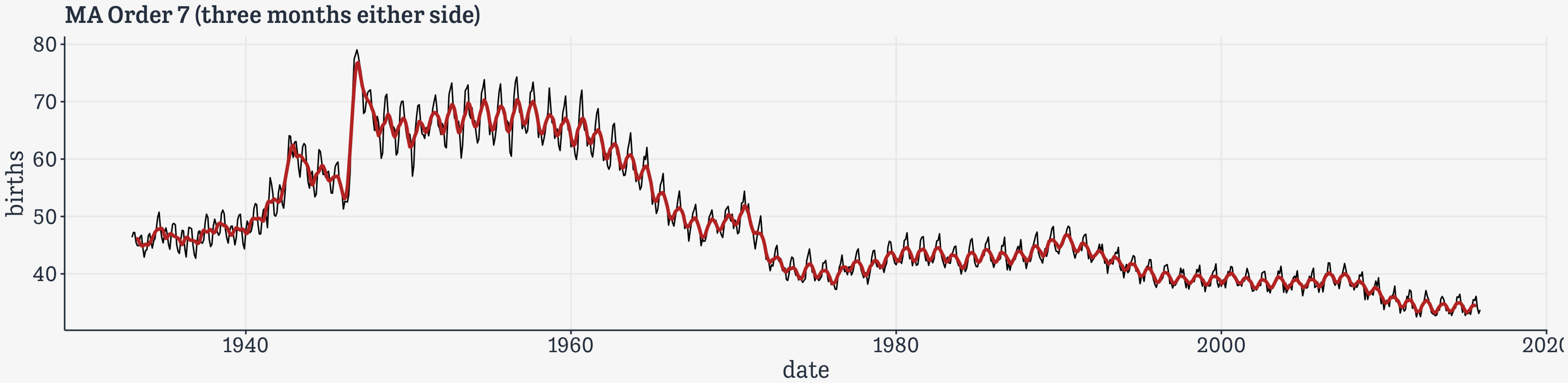


A Centered Moving Average, order 7

```r
boom |>
  mutate(
    mav = slide_dbl(births, mean,
                    .before = 3, .after = 3,
                    .complete = TRUE)) |>
  ggplot() +
  geom_line(aes(x = date, y = births)) +
  geom_line(aes(x = date, y = mav), linewidth = rel(1.2), color = "firebrick") +
  labs(title = "MA Order 7 (three months either side)")
```

As the order goes up, the window for the average widens, and the line gets smoother and smoother.

A Centered Moving Average, order 7



Odd vs Even Centering

Even orders have to be calculated differently

When the period m is odd, the average d for an observation y at a particular time t is:

$$d_t = \frac{1}{m} \sum_{i=-(m-1)/2}^{(m-1)/2} y_{t+i}$$

Odd vs Even Centering

When the period is even, it's:

$$d_t = \frac{1}{m}\left(\frac{1}{2}\left(y_{t+(m-1)/2} + y_{t-(m-1)/2}\right) + \sum_{i=-(m-2)/2}^{(m-2)/2} y_{t+i}\right)$$

This just means e.g. we use half of December of the previous year and half of December of the current year to calculate the centred moving average in June of the current year.

A Centered Moving Average of order 12

We can calculate the CMA for even orders in two steps.

```
boom |>
  mutate(
    mav12 = slide_dbl(births, mean,
                    .before = 5, .after = 6,
                    .complete = TRUE),
    mav2x12 = slide_dbl(mav12, mean,
                       .before = 1, .after = 0,
                       .complete = TRUE))
```

```
# A tibble: 996 × 5
   date       total_pop births mav12 mav2x12
   <date>         <dbl>  <dbl> <dbl>   <dbl>
 1 1933-01-01 125579000   46.4  NA      NA
 2 1933-02-01 125579000   47.2  NA      NA
 3 1933-03-01 125579000   47.2  NA      NA
 4 1933-04-01 125579000   45.5  NA      NA
 5 1933-05-01 125579000   44.9  NA      NA
 6 1933-06-01 125579000   44.9  45.4    NA
 7 1933-07-01 125579000   46.5  45.4    45.4
 8 1933-08-01 125579000   46.7  45.4    45.4
 9 1933-09-01 125579000   44.5  45.3    45.4
10 1933-10-01 125579000   42.9  45.2    45.3
# ℹ 986 more rows
```

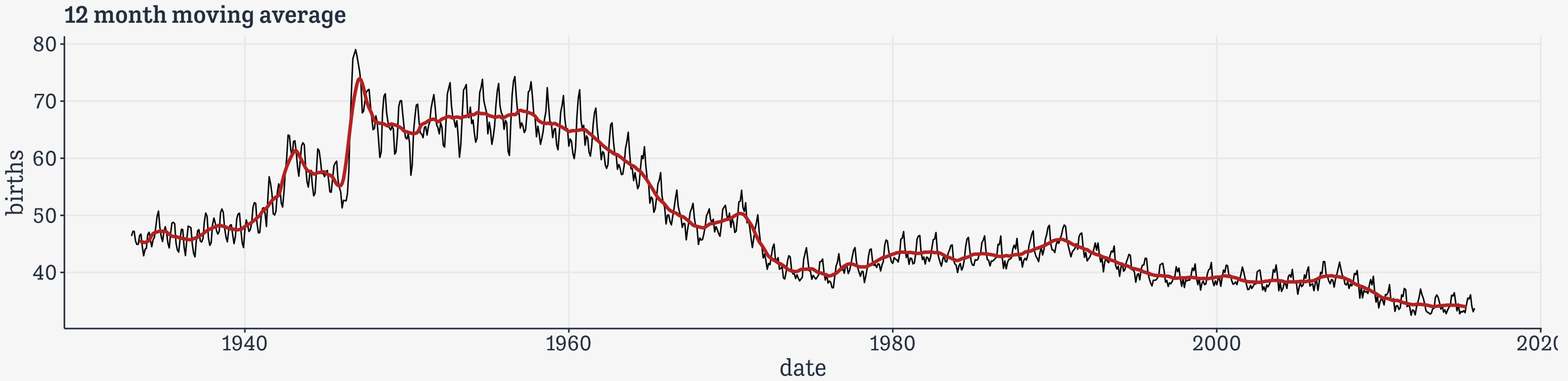
See how we lose observations as our window widens.

A Centered Moving Average, order 12

```
boom ▷
  mutate(
    mav12 = slide_dbl(births, mean,
                     .before = 5, .after = 6,
                     .complete = TRUE),
    mav2x12 = slide_dbl(mav12, mean,
                          .before = 1, .after = 0,
                          .complete = TRUE)) ▷
  ggplot() +
  geom_line(aes(x = date, y = births)) +
  geom_line(aes(x = date, y = mav2x12), linewidth = rel(1.2), color = "firebrick") +
  labs(title = "12 month moving average")
```

Doing it this way—e.g. taking a yearly average of 12 monthly values—means all the seasonality is averaged away.

A Centered Moving Average, order 12



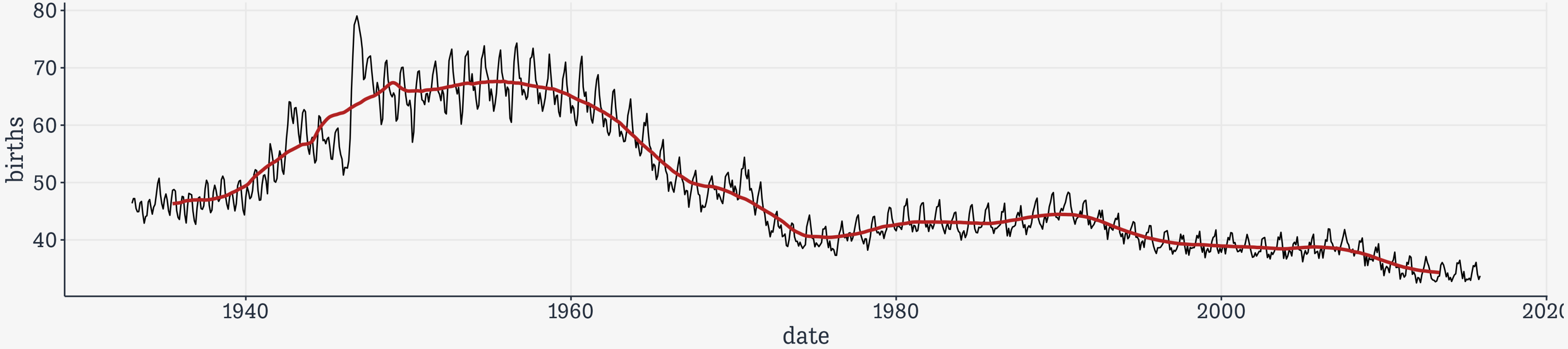
A 5 year CMA

```r
boom |>
  mutate(
    mav12 = slide_dbl(births, mean,
                .before = 29, .after = 30,
                .complete = TRUE),
    mav2x12 = slide_dbl(mav12, mean,
                .before = 1, .after = 0,
                .complete = TRUE)) |>
  ggplot() +
  geom_line(aes(x = date, y = births)) +
  geom_line(aes(x = date, y = mav2x12), linewidth = rel(1.2), color = "firebrick") +
  labs(title = "5 year moving average")
```

The wider the window, the flatter the line. And we continue to lose observations.

A 5 year CMA

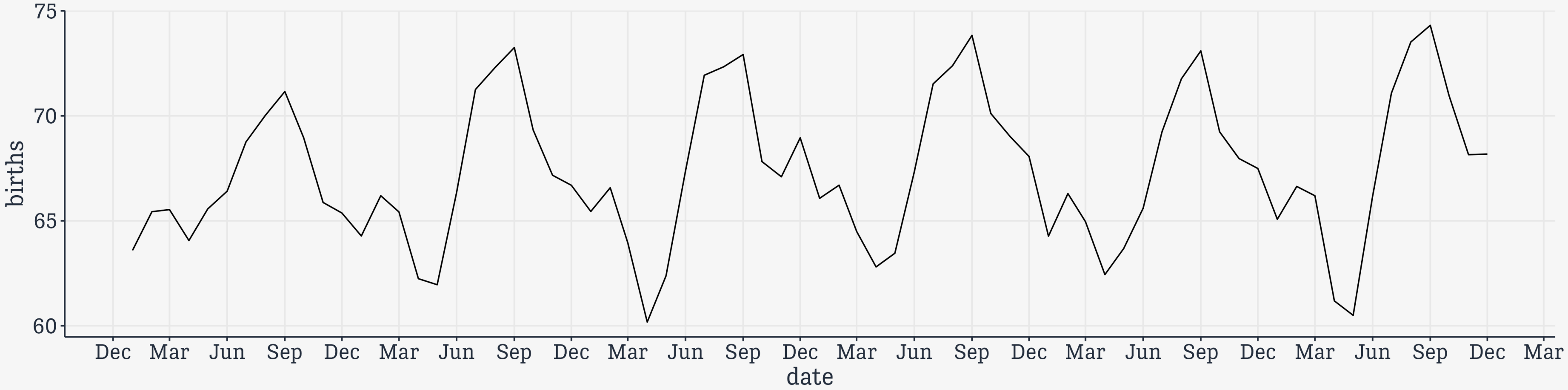
5 year moving average



Seasonality in US Birth Rates

```
boom ▷
  filter(date > as.Date("1950-12-01"),
         date < as.Date("1957-01-01")) ▷
  ggplot() +
  geom_line(aes(x = date, y = births)) +
  scale_x_date(date_breaks = "3 months",
               date_labels = "%b")
```

Seasonality in US Birth Rates



The Additive “Classical” Decomposition

The Thing to be Decomposed: the births series, y

A *Trend* Part: a centered moving average, $\hat{T}$

A *Seasonal* Part: the “pulse” in the data, $\hat{S}$

A *Remainder* Part: the leftover part, $\hat{R}$

The trend y is then $y = \hat{T} + \hat{S} + \hat{R}$

Calculate the Trend part

This is the moving average we just calculated.

```
boom_t <- boom |>
  select(date, births) |>
  mutate(
    month = lubridate::month(date),
    mav12 = slide_dbl(births, mean, .before = 5, .after = 6,
                      .complete = TRUE),
    t = slide_dbl(mav12, mean, .before = 1, .after = 0,
                       .complete = TRUE)) |>
  select(-mav12) # Don't need this anymore
boom_t
```

```
# A tibble: 996 × 4
   date       births month     t
   <date>      <dbl> <dbl> <dbl>
 1 1933-01-01   46.4     1  NA  
 2 1933-02-01   47.2     2  NA  
 3 1933-03-01   47.2     3  NA  
 4 1933-04-01   45.5     4  NA  
 5 1933-05-01   44.9     5  NA  
 6 1933-06-01   44.9     6  NA  
 7 1933-07-01   46.5     7  45.4
 8 1933-08-01   46.7     8  45.4
 9 1933-09-01   44.5     9  45.4
10 1933-10-01   42.9    10  45.3
# ℹ 986 more rows
```

Calculate the Seasonal part

First "detrend" the series by subtracting `t` from `births`.

```
boom_t ▷
  mutate(detrended = births - t)
```

```
# A tibble: 996 × 5
   date       births month     t detrended
   <date>      <dbl> <dbl> <dbl>     <dbl>
 1 1933-01-01   46.4     1  NA      NA
 2 1933-02-01   47.2     2  NA      NA
 3 1933-03-01   47.2     3  NA      NA
 4 1933-04-01   45.5     4  NA      NA
 5 1933-05-01   44.9     5  NA      NA
 6 1933-06-01   44.9     6  NA      NA
 7 1933-07-01   46.5     7  45.4     1.04
 8 1933-08-01   46.7     8  45.4     1.29
 9 1933-09-01   44.5     9  45.4    -0.861
10 1933-10-01   42.9    10  45.3    -2.34
# ℹ 986 more rows
```

Calculate the Seasonal part

Then take the average by month.

```
boom_t |>
  mutate(detrended = births - t,
         month = lubridate::month(date)) |>
  group_by(month) |>
  summarize(seasonal = mean(detrended, na.rm = TRUE))
```

```
# A tibble: 12 × 2
   month seasonal
   <dbl>    <dbl>
 1     1   -1.62
 2     2   -0.578
 3     3   -0.912
 4     4   -2.21
 5     5   -1.99
 6     6   -0.333
 7     7    1.90
 8     8    2.74
 9     9    3.39
10    10    0.765
11    11   -0.564
12    12   -0.625
```

Calculate the **Seasonal** part

Then "mean-center" each point by taking the average again and subtracting it from each observation. (This way the observations all sum to zero.)

```r
boom_t ▷
  mutate(detrended = births - t) ▷
  group_by(month) ▷
  summarize(sm = mean(detrended, na.rm = TRUE)) ▷
  mutate(s = sm - mean(sm))
```

```
# A tibble: 12 × 3
   month     sm      s
   <dbl>  <dbl>  <dbl>
 1     1 -1.62  -1.62
 2     2 -0.578 -0.575
 3     3 -0.912 -0.909
 4     4 -2.21  -2.21
 5     5 -1.99  -1.98
 6     6 -0.333 -0.330
 7     7  1.90   1.90
 8     8  2.74   2.74
 9     9  3.39   3.40
10    10  0.765  0.768
11    11 -0.564 -0.561
12    12 -0.625 -0.622
```

Calculate the Seasonal part

Put this in an object

```r
boom_s <- boom_t |>
  mutate(detrended = births - t) |>
  group_by(month) |>
  summarize(sm = mean(detrended, na.rm = TRUE)) |>
  mutate(s = sm - mean(sm)) |>
  select(-sm) # don't need this anymore

boom_s
```

```
# A tibble: 12 × 2
   month      s
   <dbl>  <dbl>
 1     1 -1.62
 2     2 -0.575
 3     3 -0.909
 4     4 -2.21
 5     5 -1.98
 6     6 -0.330
 7     7  1.90
 8     8  2.74
 9     9  3.40
10    10  0.768
11    11 -0.561
12    12 -0.622
```

Calculate the Seasonal part

Join it to the main table

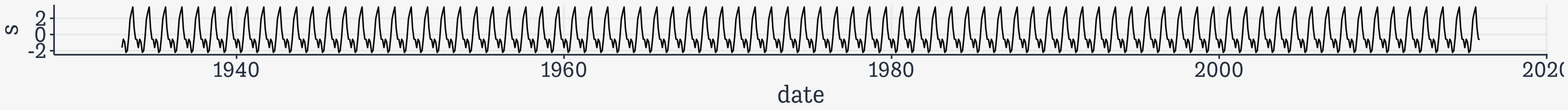
```
boom_ts <- boom_t |>
  left_join(boom_s, by = "month")

boom_ts
```

```
# A tibble: 996 × 5
   date       births month     t      s
   <date>      <dbl> <dbl> <dbl>  <dbl>
 1 1933-01-01   46.4     1  NA   -1.62
 2 1933-02-01   47.2     2  NA   -0.575
 3 1933-03-01   47.2     3  NA   -0.909
 4 1933-04-01   45.5     4  NA   -2.21
 5 1933-05-01   44.9     5  NA   -1.98
 6 1933-06-01   44.9     6  NA   -0.330
 7 1933-07-01   46.5     7  45.4  1.90
 8 1933-08-01   46.7     8  45.4  2.74
 9 1933-09-01   44.5     9  45.4  3.40
10 1933-10-01   42.9    10  45.3  0.768
# ℹ 986 more rows
```

Calculate the Seasonal part

In a “Classical” decomposition the Seasonal part just repeats.

```
boom_ts |>
  ggplot(aes(x = date, y = s)) +
  geom_line()
```



Calculate the Remainder part

The remainder is just what's left over from `y` (i.e., births) after we have calculated `t` and `s`.

```
boom_tsr <- boom_ts |>
  mutate(r = births - t - s)

boom_tsr
```

```
# A tibble: 996 × 6
   date       births month     t      s      r
   <date>      <dbl> <dbl> <dbl>  <dbl>  <dbl>
 1 1933-01-01   46.4     1  NA   -1.62  NA
 2 1933-02-01   47.2     2  NA   -0.575 NA
 3 1933-03-01   47.2     3  NA   -0.909 NA
 4 1933-04-01   45.5     4  NA   -2.21  NA
 5 1933-05-01   44.9     5  NA   -1.98  NA
 6 1933-06-01   44.9     6  NA   -0.330 NA
 7 1933-07-01   46.5     7  45.4  1.90  -0.863
 8 1933-08-01   46.7     8  45.4  2.74  -1.46
 9 1933-09-01   44.5     9  45.4  3.40  -4.26
10 1933-10-01   42.9    10  45.3  0.768 -3.11
# ℹ 986 more rows
```

Decomposition: `y = t + s + r`

This is an *additive* decomposition. You can also do *multiplicative* decompositions.

```r
boom_tsr |>
  mutate(tsr = t + s + r)
```

```
# A tibble: 996 × 7
   date       births month     t      s      r   tsr
   <date>      <dbl> <dbl> <dbl>  <dbl>  <dbl> <dbl>
 1 1933-01-01   46.4     1  NA   -1.62  NA      NA  
 2 1933-02-01   47.2     2  NA   -0.575 NA      NA  
 3 1933-03-01   47.2     3  NA   -0.909 NA      NA  
 4 1933-04-01   45.5     4  NA   -2.21  NA      NA  
 5 1933-05-01   44.9     5  NA   -1.98  NA      NA  
 6 1933-06-01   44.9     6  NA   -0.330 NA      NA  
 7 1933-07-01   46.5     7  45.4  1.90  -0.863  46.5
 8 1933-08-01   46.7     8  45.4  2.74  -1.46   46.7
 9 1933-09-01   44.5     9  45.4  3.40  -4.26   44.5
10 1933-10-01   42.9    10  45.3  0.768 -3.11   42.9
# ℹ 986 more rows
```

There's no need to it manually

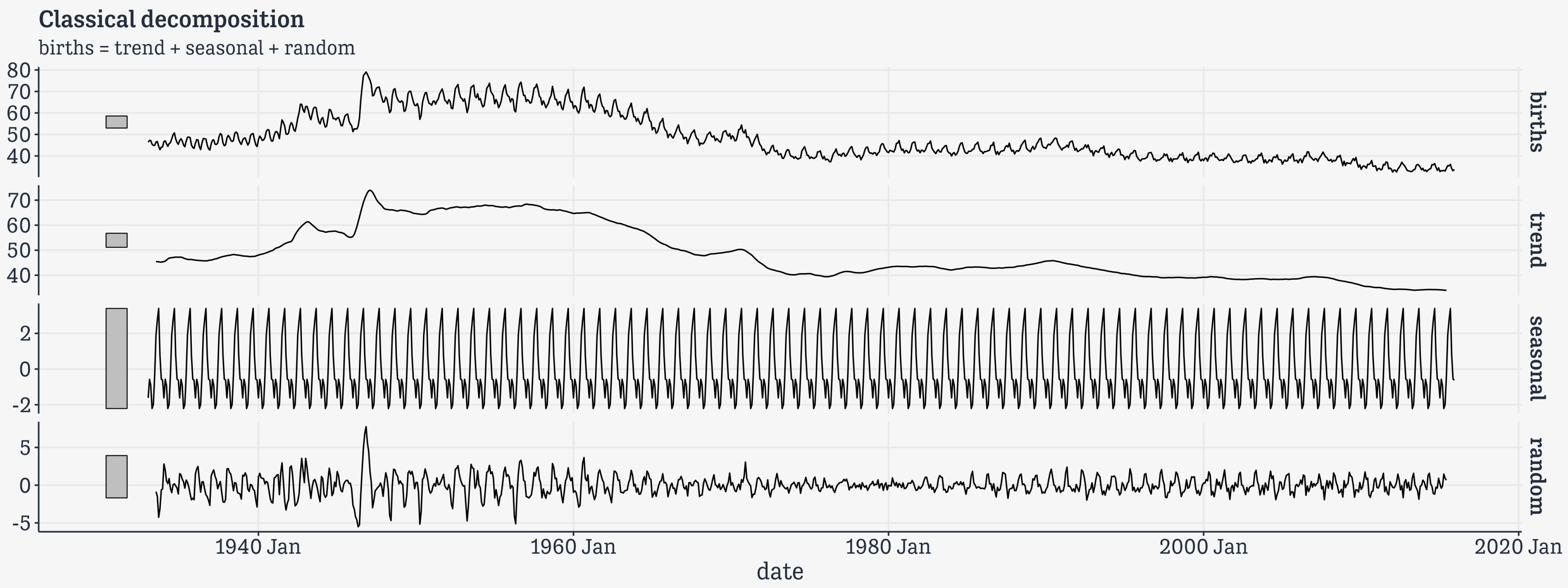
```r
boom ▷
  mutate(date = yearmonth(date)) ▷
  as_tsibble(index = "date") ▷
  model(
    classical_decomposition(births,
                            type = "additive")
  ) ▷
  components() ▷
  select(-.model)
```

```
# A tsibble: 996 x 6 [1M]
       date births trend seasonal random season_adjust
      <mth>  <dbl> <dbl>    <dbl>  <dbl>         <dbl>
 1 1933 Jan   46.4  NA     -1.62   NA             48.0
 2 1933 Feb   47.2  NA     -0.575 NA             47.8
 3 1933 Mar   47.2  NA     -0.909 NA             48.1
 4 1933 Apr   45.5  NA     -2.21   NA             47.7
 5 1933 May   44.9  NA     -1.98   NA             46.9
 6 1933 Jun   44.9  NA     -0.330 NA             45.3
 7 1933 Jul   46.5  45.4    1.90   -0.863         44.6
 8 1933 Aug   46.7  45.4    2.74   -1.46          44.0
 9 1933 Sep   44.5  45.4    3.40   -4.26          41.1
10 1933 Oct   42.9  45.3    0.768 -3.11          42.1
# ℹ 986 more rows
```

Plot all the components at once

```
boom ▷
  mutate(date = yearmonth(date)) ▷
  as_tsibble(index = "date") ▷
  model(
    classical_decomposition(births, type = "additive")
  ) ▷
  components() ▷
  autoplot()
```

Plot all the components at once



The STL Decomposition

More robust and flexible than Classical Decomposition

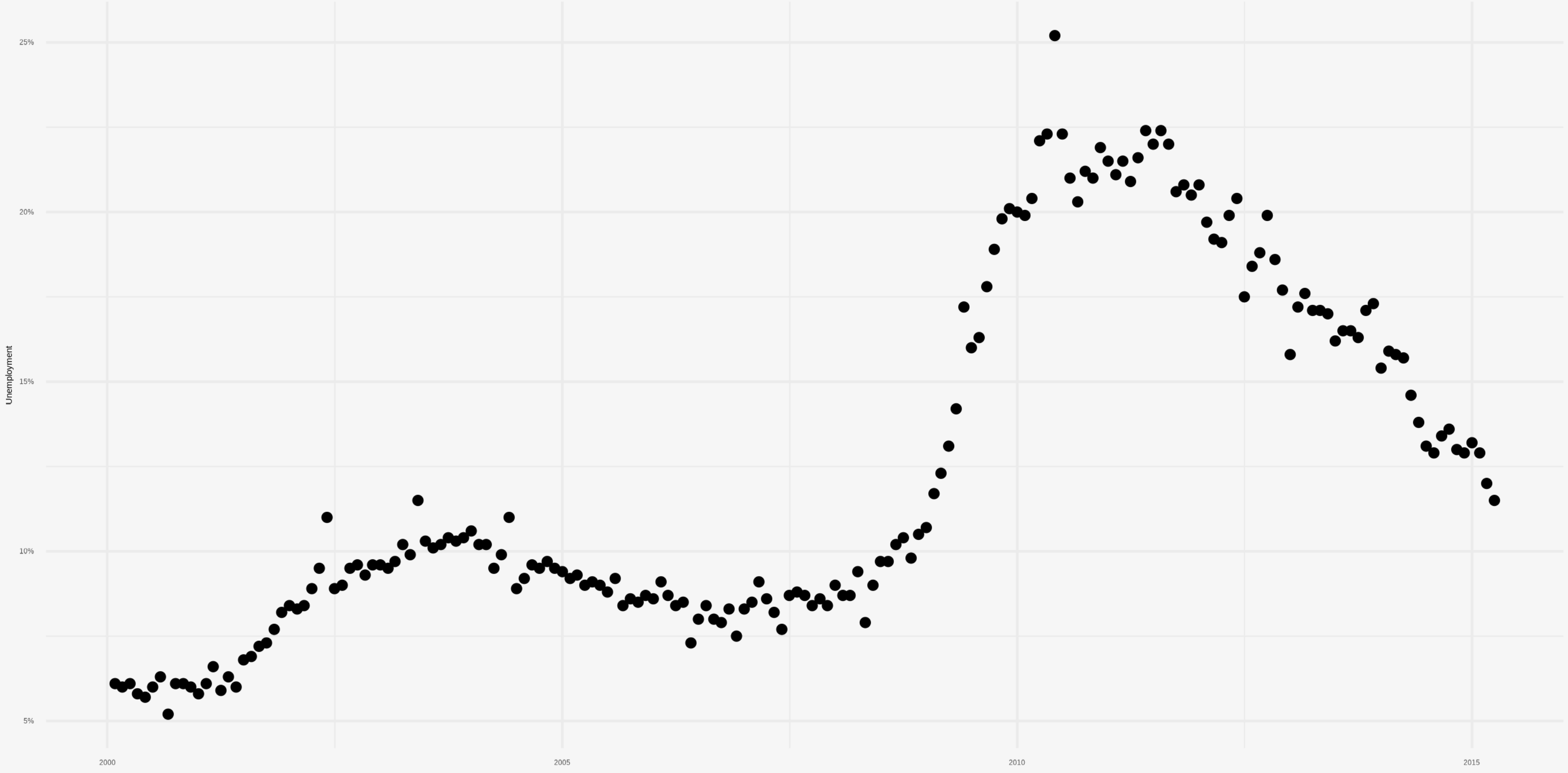
Due to William Cleveland

Uses LOESS, a little like `geom_smooth()`

Good for monthly and annual data

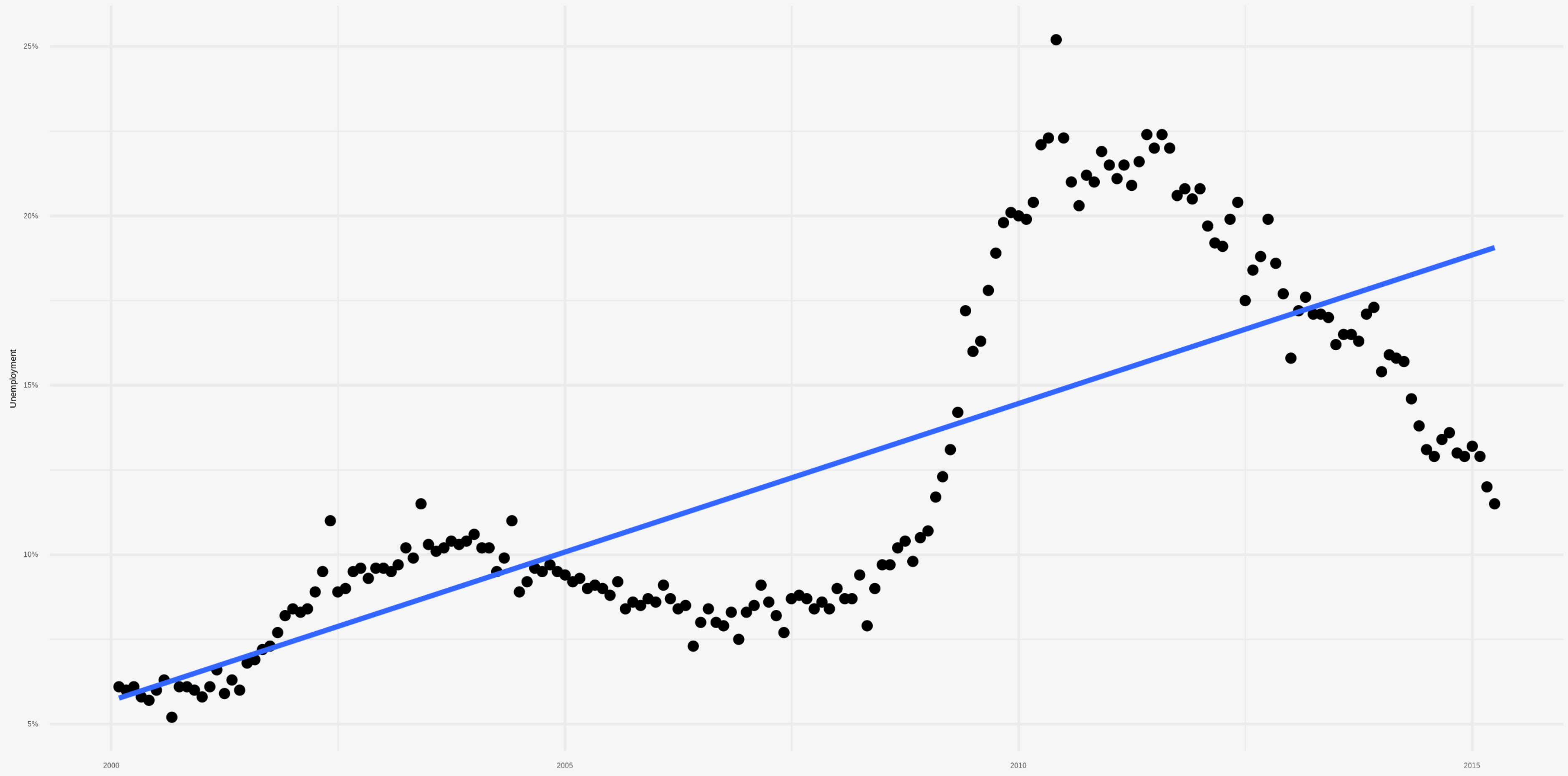
You have to choose the Trend and Seasonal Windows

Sidenote: Smoothers



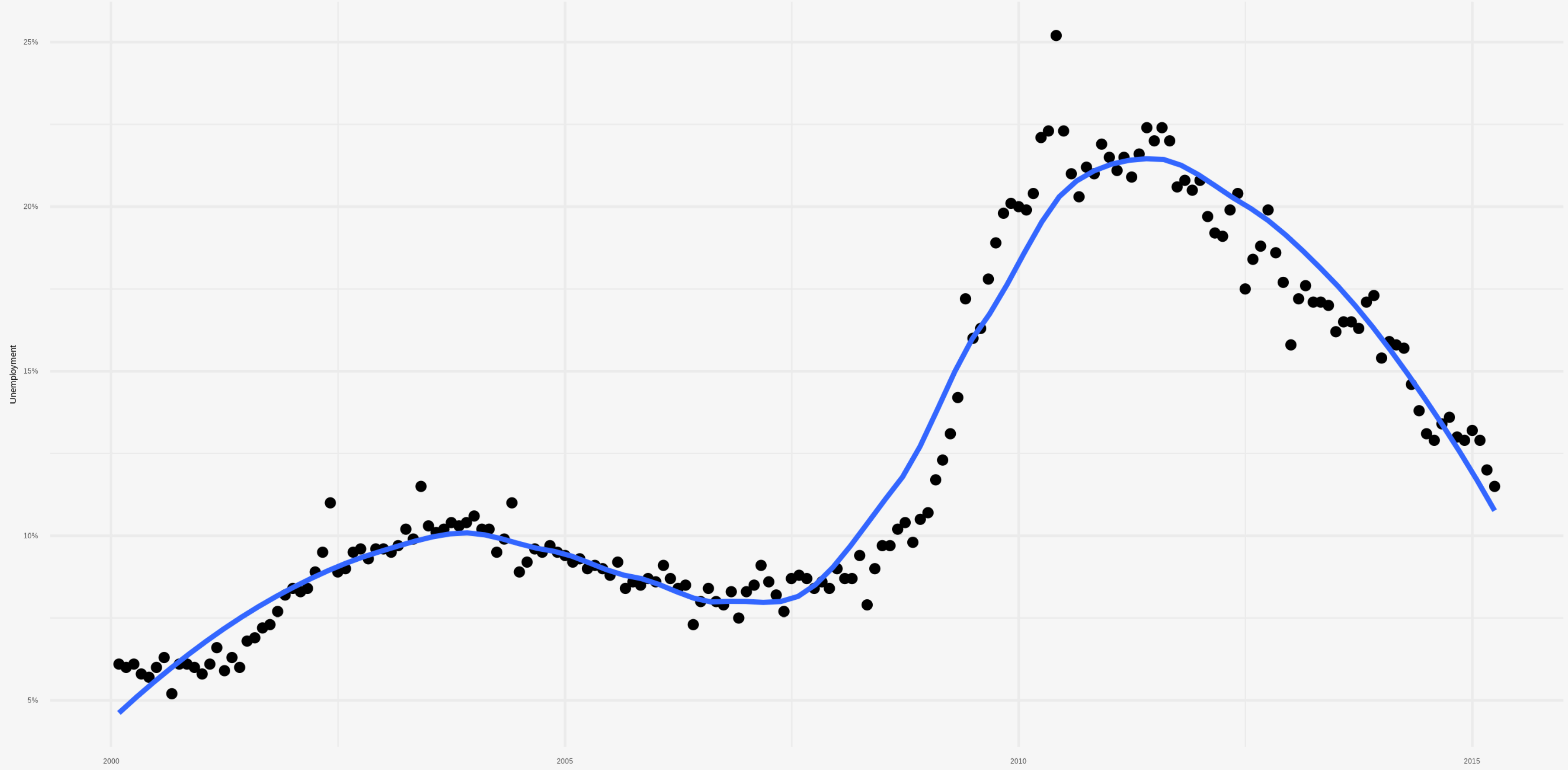
A trend

Sidenote: Smoothers



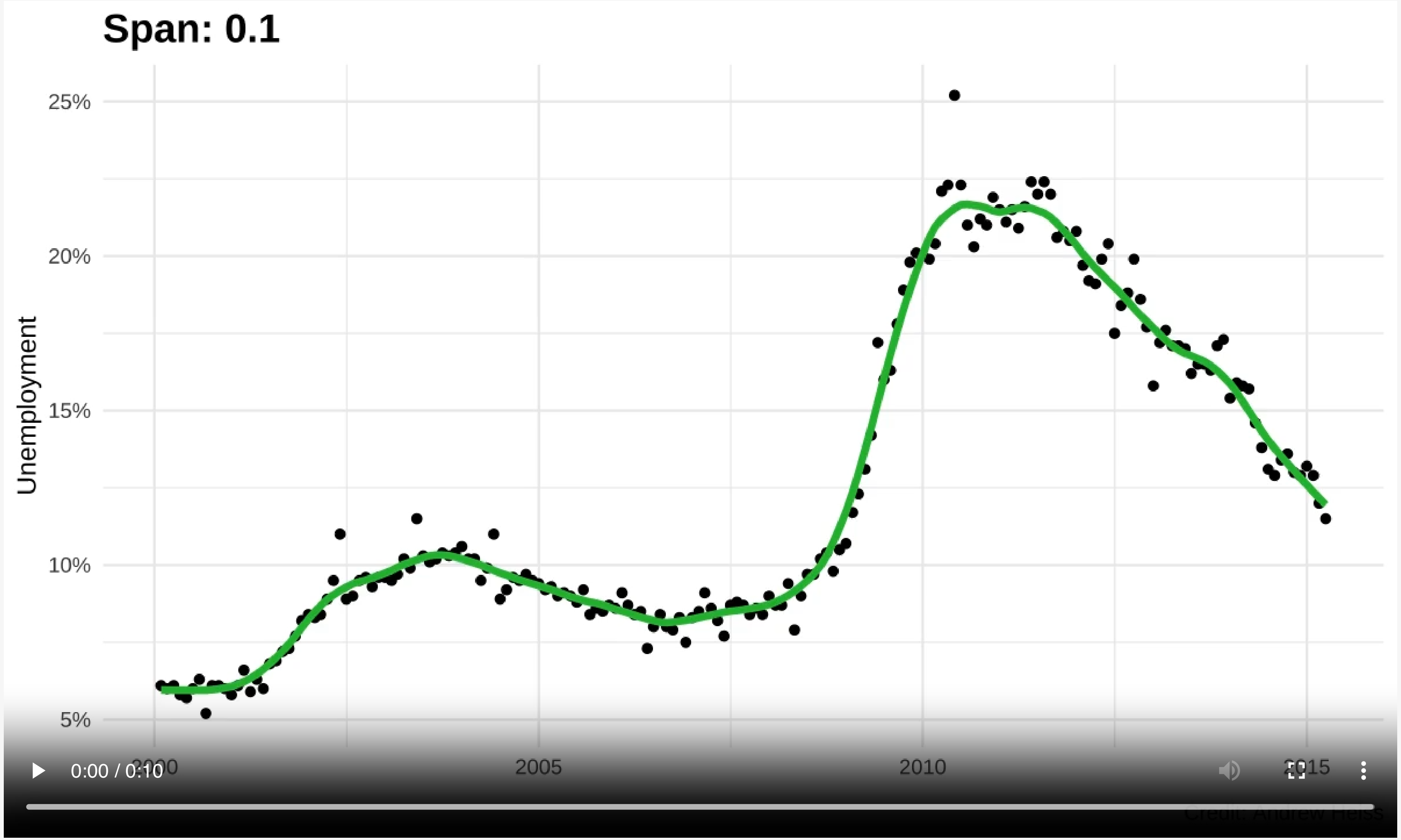
Smoother with bad linear fit

Sidenote: Smoothers

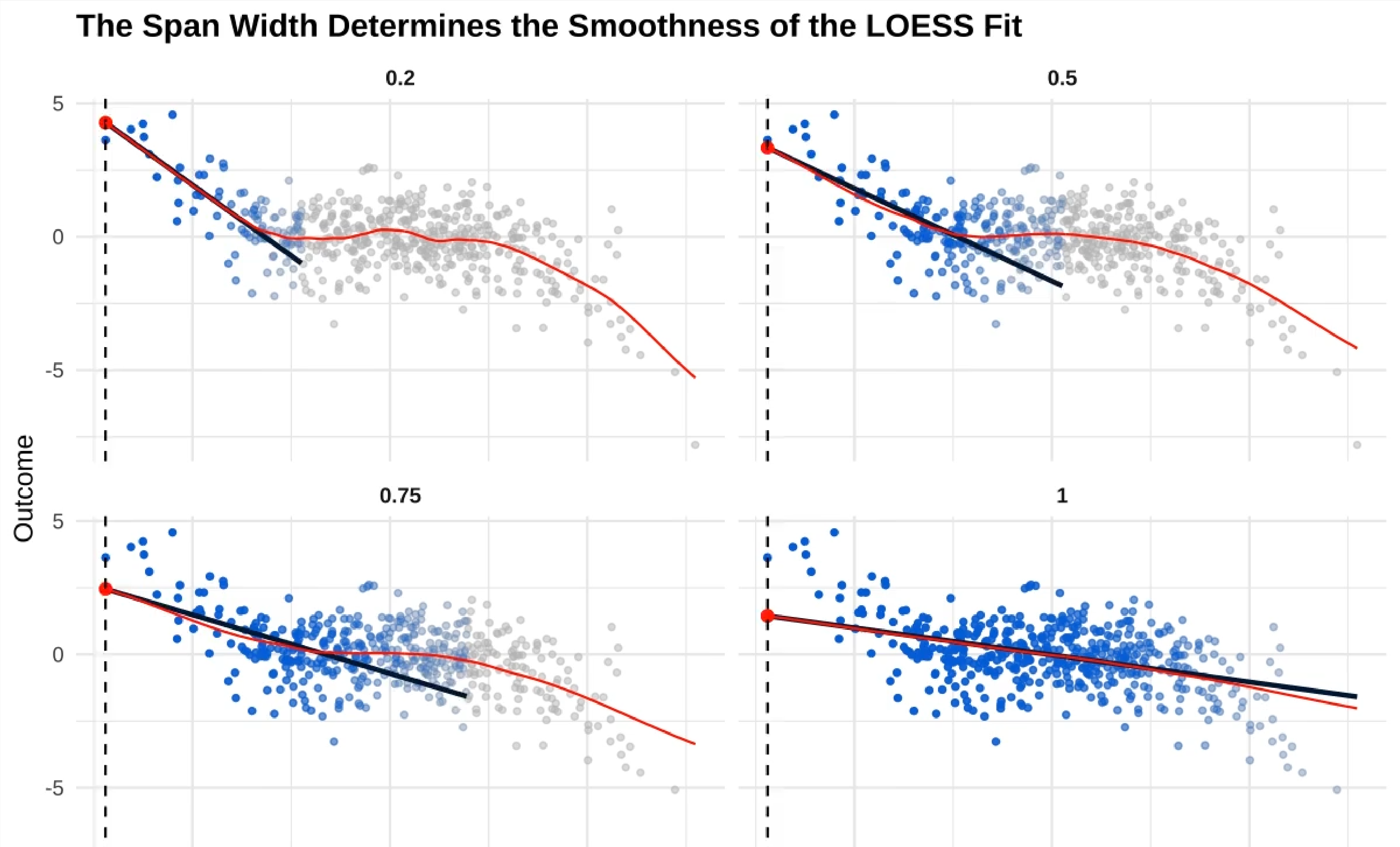


Smoother with loess fit

Sidenote: Smoothers



Sidenote: Smoothers



The STL Decomposition

Default seasonal monthly window is 13

This works for monthly data

Default monthly trend window is 21

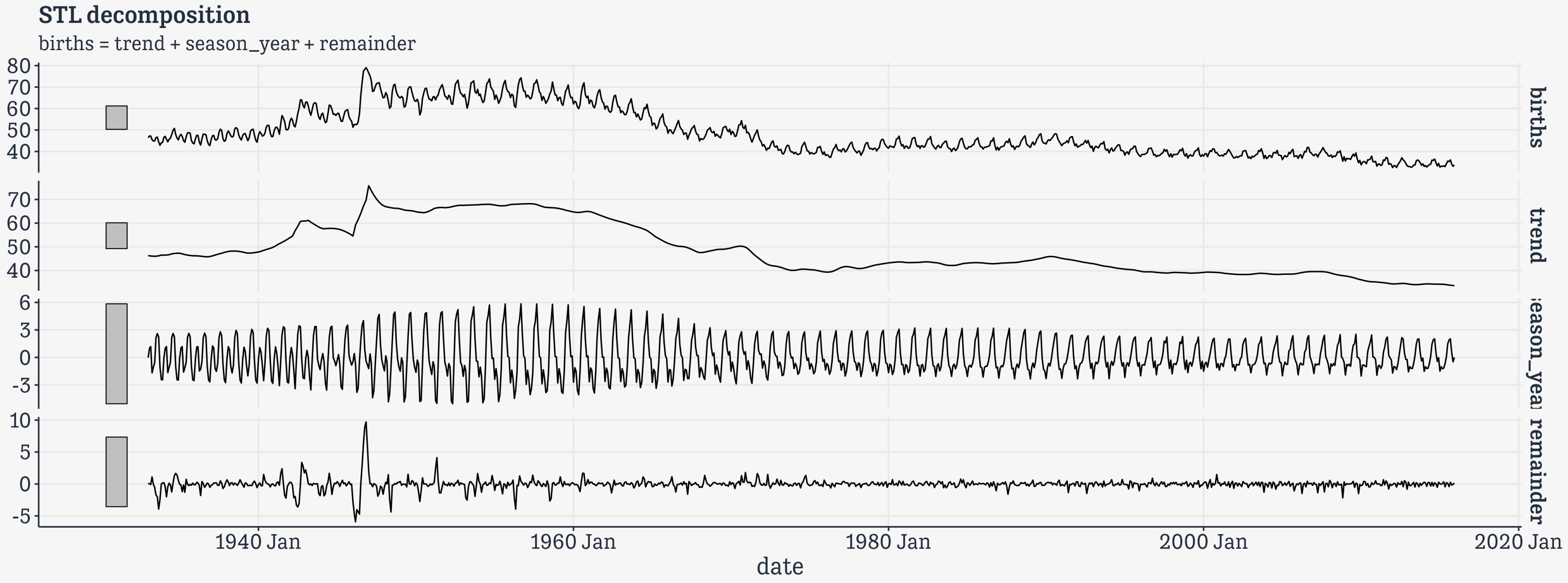
You can experiment with these

They should be odd numbers

The STL Decomposition

```r
boom |>
  mutate(date = yearmonth(date)) |>
  as_tsibble(index = "date") |>
  model(
    STL(births ~ trend(window = 13) +
          season(window = 7),
        robust = TRUE)
  ) |>
  components() |>
  autoplot()
```

The STL Decomposition



Manually plotting the components

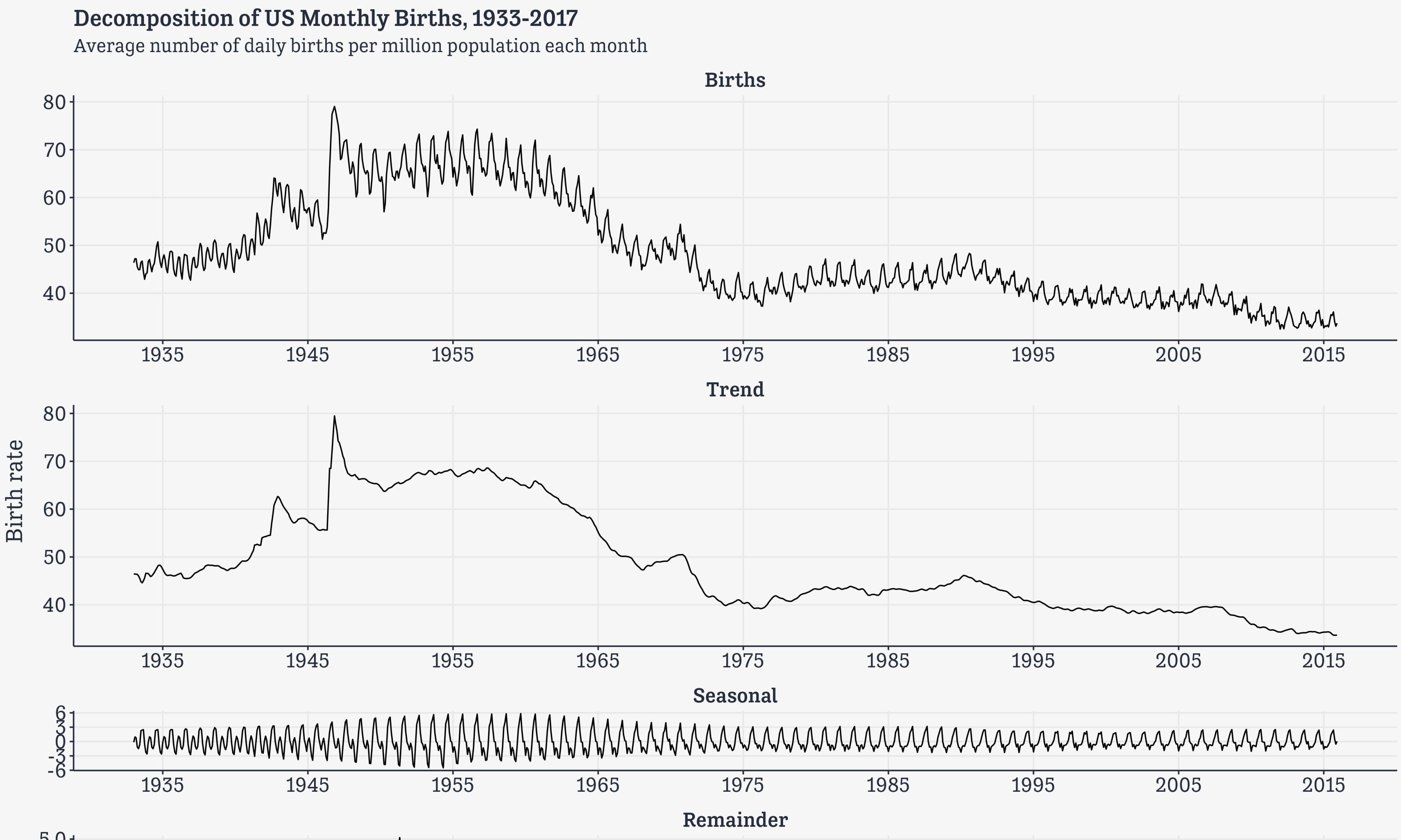
```r
bc <- boom |>
  mutate(date = yearmonth(date)) |>
  as_tsibble(index = "date") |>
  model(
    # Experiment with a six-monthly trend window
    STL(births ~ trend(window = 7) +
          season(window = 7),
        robust = TRUE)
  ) |>
  components() |>
  select(-.model)

bc
```

```
# A tsibble: 996 x 6 [1M]
       date births trend season_year remainder season_adjust
      <mth>  <dbl> <dbl>       <dbl>     <dbl>         <dbl>
 1 1933 Jan   46.4  46.4      -0.148    0.0757          46.5
 2 1933 Feb   47.2  46.4       0.969   -0.197           46.2
 3 1933 Mar   47.2  46.4       0.781    0.0247          46.4
 4 1933 Apr   45.5  46.3      -1.17     0.304           46.7
 5 1933 May   44.9  46.1      -1.49     0.329           46.4
 6 1933 Jun   44.9  45.5      -0.646    0.0632          45.6
 7 1933 Jul   46.5  44.8       2.25    -0.632           44.2
 8 1933 Aug   46.7  44.6       2.40    -0.278           44.3
 9 1933 Sep   44.5  45.0       2.47    -2.98            42.0
10 1933 Oct   42.9  45.6      -0.584   -2.12            43.5
```

Manually plotting the components

```r
bc ▷
  pivot_longer(cols = c(births, trend, season_year, remainder)) ▷
  mutate(
    date = as.Date(date),
    name = factor(name, levels = c("births", "trend",
                                    "season_year", "remainder"),
                  labels = c("Births", "Trend", "Seasonal", "Remainder"),
                       ordered = TRUE)) ▷
  ggplot() +
  geom_line(aes(date, value)) +
  scale_x_date(breaks = seq(as.Date("1935-01-01"),
                             as.Date("2015-01-01"),
                             by="10 years"),
               date_labels = "%Y") +
  ggforce::facet_col(~ name, scales = 'free', space = 'free') +
  labs(title = "Decomposition of US Monthly Births, 1933-2017",
       subtitle = "Average number of daily births per million population each month",
       x = "Time", y = "Birth rate")
```

Manually plotting the components



Comparing seasonality

```r
s30s <- lubridate::interval(ymd(19330101), ymd(19390101))
s50s <- lubridate::interval(ymd(19530101), ymd(19590101))
s00s <- lubridate::interval(ymd(20030101), ymd(20090101))

my_intervs <- list(s30s, s50s, s00s)

bc_int <- bc |>
  mutate(date = as.Date(date)) |>
  filter(date %within% my_intervs) |>
  mutate(period = case_when(
    date %within% s30s ~ "1930s",
    date %within% s50s ~ "1950s",
    date %within% s00s ~ "2000s"),
    year = year(date),
    month = month(date, label = TRUE,
                  abbr = TRUE)) |>
  mutate(yr_id = consecutive_id(year), .by = period) |>
  mutate(mth_id = row_number(), .by = c(period, year)) |>
  mutate(seq_id = row_number(), .by = period)
```

Comparing seasonality

```r
bc_int |>
  print(n = 20)
```

```
# A tsibble: 219 x 12 [1D]
   date       births trend season_year remainder season_adjust period  year
   <date>      <dbl> <dbl>       <dbl>     <dbl>         <dbl> <chr>  <dbl>
 1 1933-01-01   46.4  46.4      -0.148    0.0757          46.5 1930s   1933
 2 1933-02-01   47.2  46.4       0.969   -0.197           46.2 1930s   1933
 3 1933-03-01   47.2  46.4       0.781    0.0247          46.4 1930s   1933
 4 1933-04-01   45.5  46.3      -1.17     0.304           46.7 1930s   1933
 5 1933-05-01   44.9  46.1      -1.49     0.329           46.4 1930s   1933
 6 1933-06-01   44.9  45.5      -0.646    0.0632          45.6 1930s   1933
 7 1933-07-01   46.5  44.8       2.25    -0.632           44.2 1930s   1933
 8 1933-08-01   46.7  44.6       2.40    -0.278           44.3 1930s   1933
 9 1933-09-01   44.5  45.0       2.47    -2.98            42.0 1930s   1933
10 1933-10-01   42.9  45.6      -0.584   -2.12            43.5 1930s   1933
11 1933-11-01   44.0  46.6      -2.24    -0.298           46.3 1930s   1933
12 1933-12-01   44.2  46.6      -2.60     0.247           46.8 1930s   1933
13 1934-01-01   46.6  46.5      -0.159    0.272           46.8 1930s   1934
14 1934-02-01   47.0  46.2       0.988   -0.167           46.1 1930s   1934
15 1934-03-01   45.7  45.9       0.791   -0.984           44.9 1930s   1934
16 1934-04-01   44.5  46.1      -1.17    -0.397           45.7 1930s   1934
```

Comparing seasonality

```r
my_labs <- bc_int$seq_id
names(my_labs) <- bc_int$month

ind <- names(my_labs) %in% c("Jan", "May", "Sep")

my_labs <- my_labs[ind]

bc_int |>
  ggplot(aes(x = seq_id,
             y = season_year,
             color = period)) +
  geom_line(linewidth = rel(1.2)) +
  scale_x_continuous(breaks = my_labs,
                     labels = names(my_labs)) +
  facet_wrap(~ period, ncol = 1) +
  guides(color = "none") +
  labs(x = "Month",  y = "Seasonal Component of the Birth Rate",
       title = "Changing Seasonality in Births: Three Six-Year periods in Three Decades",
       subtitle = "Seasonal Component from an STL decomposition of 1933-2015 monthly births")
```

Comparing seasonality